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Modal Multimodel Control Design Approach Applied to Aircraft Autopilot Design

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Modal approaches such as eigenstructure assignment have shown themselves to be efficient for flight control design. Performance requirements are easily met using this approach. However, generally, robustness is not satisfactory. A technique is presented that can be viewed as an improvement over traditional eigenstructure assignment as it produces systems that meet robustness requirements (multimodel approach). The proposed technique reduces to solving a quadratic problem under linear constraints. The application treated concerns the landing phase of a large transport aircraft. It is shown that standard gain scheduling can be replaced by a single low-dimensional dynamic feedback.

I. Introduction

SINCE the initial contributions,[1,2] eigenstructure assignment has received much attention, especially for aerospace applications. Now these techniques are used for flight control of several civil aircraft.[3,4] References 1–4 and also, for example, Refs. 5–7 deal mainly with shaping the time response by assigning to zero some entries of the right eigenvectors (decoupling). Alternative techniques consider eigenvalue insensitivity.[8,9] For the design problem considered here, insensitivity is not sufficient, because parameter variations are too large to permit a first-order treatment; also, standard decoupling requirements are difficult to meet.

Simultaneous treatment of robustness and decoupling by eigenstructure assignment is considered in several references.[10–14] In each case, it is necessary to use iterative techniques as nonconvex optimization. The approach proposed in this paper is based on solutions that can be computed without iterations (quadratic optimization problem).

Dynamic regulators were already considered to handle more degrees of freedom so that robustness can be improved.[15] This paper presents a new point of view for using the additional freedom offered by dynamic control. The main advantage lies in the possible treatment of multimodel design. It is also possible to consider direct measurement feedback and structured gain control using the proposed approach.

In the aforementioned references, the gain, a constant matrix, results from a set of linear constraints. In this paper, the same constraints are considered but the gain is a transfer function matrix evaluated at the assigned poles. Such an equation is more difficult to solve with dynamics than without, but an additional difficulty arises from the fact that often dynamics introduce too many degrees of freedom. A specific description of the degrees of freedom is proposed that, to deal with free parameter redundancy, enables the designer to define meaningful quadratic criteria. The resulting design procedure turns out to be a simple problem of quadratic optimization under linear equality constraints. Unlike Ref. 15, the stability of the poles introduced by the denominator of the dynamic feedback are not directly treated here. In practice, no problems are encountered if care is exercised, and the design procedure described in Sec. IV prevents such problems.

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The proposed technique is illustrated considering the design of the longitudinal autopilot of a large transport aircraft.[16] When such a design is based on eigenstructure assignment, gain scheduling with respect to at least one of the varying parameters is necessary. It is shown that the simple design procedure of Sec. IV produces a low-dimensional dynamic controller, which satisfies decoupling, settling time, and damping ratio requirements for all operating points corresponding to weight, vertical and horizontal position of the center of gravity, and air speed variations.

II. Principle of the Proposed Method

In this section it is shown that Eq. (5), usually used when proportional gains are considered, also holds in the dynamic feedback case. We shall consider the following linear system with n states, m inputs, and p outputs:

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u}, \qquad \boldsymbol{y} = C\boldsymbol{x} + D\boldsymbol{u} \tag{1}$$

where $\boldsymbol{x}$ is the vector of states, $\boldsymbol{y}$ the vector of measurements, and $\boldsymbol{u}$ the vector of inputs, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{p\times m}$.

Consider a dynamic output feedback $K(s)$,

$$\boldsymbol{u}(s) = K(s)\boldsymbol{y}(s) \tag{2}$$

This dynamic feedback will also be denoted

$$K(s) = C_c(sI - A_c)^{-1}B_c + D_c \tag{3}$$

in which n_c is the number of states involved in this state-space realization of the controller and $A_c \in \mathbb{R}^{n_c\times n_c}$, $B_c \in \mathbb{R}^{n_c\times p}$, $C_c \in \mathbb{R}^{m\times n_c}$, and $D_c \in \mathbb{R}^{m\times p}$.

First, let us define what is meant by eigenvector assignment by dynamic feedback. As it will appear in the proof of Proposition 1, the use of a dynamic feedback induces an extension of the state space. We shall say that $\boldsymbol{v}_i \in \mathbb{C}^n$ is assigned by $K(s)$ if there exists some vector $\boldsymbol{v}_{ic} \in \mathbb{C}^{n_c}$ such that $[\boldsymbol{v}_i^T \boldsymbol{v}_{ic}^T]^T$ is a closed-loop eigenvector; more precisely, it is as defined in Eq. (8). The following result is traditional when $K(s)$ is considered as a constant matrix K.

Proposition 1. Consider a pair $(\lambda_i, \boldsymbol{v}_i)$, $\lambda_i \in \mathbb{C}$, $\boldsymbol{v}_i \in \mathbb{C}^n$, which satisfies for some vector $\boldsymbol{w}_i \in \mathbb{C}^m$

$$[A - \lambda_i I \quad B]\begin{bmatrix}\boldsymbol{v}_i\\ \boldsymbol{w}_i\end{bmatrix} = 0 \tag{4}$$

This pair is assigned by a dynamic gain $K(s)$ if and only if

$$K(\lambda_i)(C\boldsymbol{v}_i + D\boldsymbol{w}_i) = \boldsymbol{w}_i \tag{5}$$

Proof: Consider the system of Eq. (1) controlled by the dynamic feedback $K(s)$ as in Eq. (3).

First, assume that $D = 0$: The state-space matrix of the closed-loop system is

$$\begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \tag{6}$$

the closed-loop right eigenvectors are denoted

$$\begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{v}_{ic} \end{bmatrix} \tag{7}$$

Thus,

$$\begin{bmatrix} A + BD_cC & BC_c \\ B_cC & A_c \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{v}_{ic} \end{bmatrix} = \lambda_i \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{v}_{ic} \end{bmatrix} \tag{8}$$

or

$$(A + BD_cC)\boldsymbol{v}_i + BC_c\boldsymbol{v}_{ic} = \lambda_i \boldsymbol{v}_i \tag{9}$$

$$B_cC\boldsymbol{v}_i + A_c\boldsymbol{v}_{ic} = \lambda_i \boldsymbol{v}_{ic} \tag{10}$$

Rearranging Eq. (10),

$$\boldsymbol{v}_{ic} = (\lambda_i I - A_c)^{-1} B_c C \boldsymbol{v}_i \tag{11}$$

Then $\boldsymbol{v}_{ic}$ is substituted into Eq. (9):

$$\left[A + BD_cC + BC_c(\lambda_i I - A_c)^{-1} B_c C\right]\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i \tag{12}$$

or

$$[A + BK(\lambda_i)C]\boldsymbol{v}_i = \lambda_i \boldsymbol{v}_i \tag{13}$$

Thus, we can define the vector $\boldsymbol{w}_i$ as in the constant gain case[2] as

$$\boldsymbol{w}_i = K(\lambda_i)C\boldsymbol{v}_i \tag{14}$$

and from Eq. (13) this vector satisfies

$$[A - \lambda_i I \quad B] \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{w}_i \end{bmatrix} = 0 \tag{15}$$

For the converse, assume that Eqs. (15) and (14) are satisfied, then defining $\boldsymbol{v}_{ic} = (\lambda_i I - A_c)^{-1} B_c C \boldsymbol{v}_i$, Eqs. (9) and (10) and then Eq. (8) follow, which means that $(\lambda_i, \boldsymbol{v}_i)$ is assigned.

Second, assume that $D \neq 0$: It is known that the feedback K relative to (A, B, C, D) is equivalent to the feedback $\tilde{K}$ relative to (A, B, C), where

$$\tilde{K}(\lambda_i) = [I - K(\lambda_i)D]^{-1} K(\lambda_i) \tag{16}$$

From Eq. (14), $\boldsymbol{w}_i$ is defined here as $\boldsymbol{w}_i = \tilde{K}(\lambda_i)C\boldsymbol{v}_i$. Thus, from Eq. (16)

$$\boldsymbol{w}_i = K(\lambda_i)(C\boldsymbol{v}_i + D\boldsymbol{w}_i) \tag{17}$$

In conclusion, we have shown that if $[\boldsymbol{v}_i^T \quad \boldsymbol{v}_{ic}^T]^T$ is a closed-loop right eigenvector there exists a vector $\boldsymbol{w}_i$ such that Eq. (15) is satisfied and that any dynamic gain $K(s)$ that performs this assignment satisfies the constraint (17).

For the converse, if $(\boldsymbol{v}_i, \boldsymbol{w}_i)$ satisfies Eqs. (15) and (17), there exists a vector $\boldsymbol{v}_{ic}$ such that $[\boldsymbol{v}_i^T \quad \boldsymbol{v}_{ic}^T]^T$ is a closed-loop right eigenvector.

In the proportional feedback case, Eq. (5) is completed by $K(C\bar{\boldsymbol{v}}_i + D\bar{\boldsymbol{w}}_i) = \bar{\boldsymbol{w}}_i$ to ensure that the gain matrix K is real. Here, in the dynamic feedback case, we must ensure that the coefficients of the transfer matrix $K(s)$ are real. This property is satisfied if and only if

$$K(\bar{s}) = \overline{K(s)} \tag{18}$$

Therefore, Proposition 1 should be completed by replacing Eq. (5) by

$$K(\lambda_i)(C\boldsymbol{v}_i + D\boldsymbol{w}_i) = \boldsymbol{w}_i, \qquad K(\bar{\lambda}_i)(C\bar{\boldsymbol{v}}_i + D\bar{\boldsymbol{w}}_i) = \bar{\boldsymbol{w}}_i \tag{19}$$

Under this condition there exists at least one real solution; see Eq. (26).

III. Resolution of Eq. (19)

There are many ways to solve Eq. (19). For example, the parameters of the state-space representation of $K(s)$, namely, the entries of (A_c, B_c, C_c, D_c) can be considered as being the unknowns.[17] Here, we consider $K(s)$ under a transfer matrix form, as shown in Eq. (20), where q is chosen a priori,

$$K(s) = \frac{\begin{bmatrix} b_{011} + b_{111}s + \cdots + b_{q11}s^q & \cdots & b_{01p} + b_{11p}s + \cdots + b_{q1p}s^q \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ b_{0m1} + b_{1m1}s + \cdots + b_{qm1}s^q & \cdots & b_{0mp} + b_{1mp}s + \cdots + b_{qmp}s^q \end{bmatrix}}{a_0 + a_1 s + \cdots + a_q s^q} \tag{20}$$

This form permits us to take advantage of the remaining degrees of freedom [fewer equations of the form of Eq. (5) than unknowns] by minimizing meaningful quadratic criteria. For example, it is possible to consider Frobenius norm-based criteria, such as $\|K(s_j)\|_F$, $\|K(s_j) - K_0\|_F$, $\|\alpha I - K(s_j)G(s_j)\|_F$, simultaneously for several complex numbers s_j.

Finally, when the coefficients a_i and b_{ijk} are found, we can use a minimal realization routine to obtain (A_c, B_c, C_c, D_c).

A. Linear Constraints Derivation

We have to consider the system (19) in which $K(s)$ is as in Eq. (20). Let us write $K(s)$ in the following form:

$$K(s) = \frac{X[I_p \otimes \Lambda(s)]}{\boldsymbol{Y}\Lambda(s)} \tag{21}$$

in which $\otimes$ denotes the Kronecker product and

$$\Lambda(\lambda) = \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^q \end{bmatrix} \tag{22}$$

$$X = \begin{bmatrix} b_{011} & b_{111} & \cdots & b_{q11} & \cdots & \cdots & b_{01p} & b_{11p} & \cdots & b_{q1p} \\ & & \vdots & & & & & & \vdots & \\ b_{0m1} & b_{1m1} & \cdots & b_{qm1} & \cdots & \cdots & b_{0mp} & b_{1mp} & \cdots & b_{qmp} \end{bmatrix} \tag{23}$$

$$\boldsymbol{Y} = [a_0 \quad a_1 \quad \cdots \quad a_q] \tag{24}$$

Equation (19) can be written as

$$\begin{aligned} X[I_p \otimes \Lambda(\lambda_i)](C\boldsymbol{v}_i + D\boldsymbol{w}_i) &= \boldsymbol{Y}\Lambda(\lambda_i)\boldsymbol{w}_i \\ X[I_p \otimes \Lambda(\bar{\lambda}_i)](C\bar{\boldsymbol{v}}_i + D\bar{\boldsymbol{w}}_i) &= \boldsymbol{Y}\Lambda(\bar{\lambda}_i)\bar{\boldsymbol{w}}_i \end{aligned} \tag{25}$$

or

$$\begin{aligned} X\Re\{[I_p \otimes \Lambda(\lambda_i)](C\boldsymbol{v}_i + D\boldsymbol{w}_i)\} &= Y\Re[\Lambda(\lambda_i)\boldsymbol{w}_i] \\ X\Im\{[I_p \otimes \Lambda(\lambda_i)](C\boldsymbol{v}_i + D\boldsymbol{w}_i)\} &= Y\Im[\Lambda(\lambda_i)\boldsymbol{w}_i] \end{aligned} \tag{26}$$

where $\Re$ and $\Im$ stand for real and imaginary parts. Equation (26) involves only real numbers; therefore, real coefficients (entries of X and $\boldsymbol{Y}$) can be found. This system is clearly linear in the unknowns. The coefficients of the denominator of $K(s)$ can be considered as a subset of the unknowns, but to obtain quadratic criteria, they will be assumed to be fixed. They are chosen as being the coefficients of some stable polynomial.

B. Derivation of the Quadratic Criteria

All of the criteria considered here are special cases of

$$J_j = \|M + NK(s_j)\|_F \tag{27}$$

for some matrices M and N of appropriate size. In view of Eq. (21), J_j can be written

$$J_j = \left\| M + N \frac{X[I_p \otimes \Lambda(s_j)]}{Y\Lambda(s_j)} \right\|_F \tag{28}$$

J_j depends on M and N. It may represent numerous criteria such as the norm of the gain, the norm of the difference between the gain and another fixed one, or the norm relative to sensitivity functions.[18]

If the vector $\boldsymbol{Y}$ is fixed, the criterion J_j is clearly quadratic with respect to the entries of the matrix X. In fact, we shall consider a sum of elementary criteria as stated earlier to cover a range of frequencies.

IV. Multimodel Design Procedure

A bank of linearized models covering all of the flight domain is assumed to be at the designer's disposal. The first step of the proposed design procedure consists of choosing a model (model 1) and designing a preliminary proportional feedback using traditional eigenstructure assignment.[4,5,7] All models are controlled by this feedback, and a multimodel analysis is used to detect the worst case (model 2). The second step consists of improving the behavior of model 2 while keeping good performance relative to model 1. The main objective of this section is to give details on the simultaneous treatment of models 1 and 2. (Additional steps might be necessary; simultaneous treatment of more than two models is similar.) The advantage of using projection for eigenvector assignment is discussed first.

A. Projection of Eigenvectors

Assuming that an initial gain relative to model 1 is known, to avoid bad results it is worth ensuring some continuity from the initial gain to the multimodel controller. For that purpose, orthogonal projection of right eigenvectors will be considered [see Eqs. (30) and (31)]. Indeed, when an eigenvalue is varied from its nominal value, if the corresponding eigenvector is chosen as being the orthogonal projection, abrupt changes in performance are avoided. An additional benefit of eigenvector projection is that the clouds of poles corresponding to a set of operating point models are not spread over a larger area. To illustrate this point, compare the pole maps given in Figs. 1 and 2 or in Figs. 3 and 4.

Thus, let us define the vector spaces in which closed-loop eigenvectors can be chosen. From Eq. (4), this vector space is the column span of $V(\lambda_i) \in \mathbb{C}^{n\times m}$ defined by (if λ_i is not controllable, $V(\lambda_i) \in \mathbb{C}^{n\times m'}$ with $m' > m$)

$$[A - \lambda_i I \quad B]\begin{bmatrix} V(\lambda_i) \\ W(\lambda_i) \end{bmatrix} = 0 \tag{29}$$

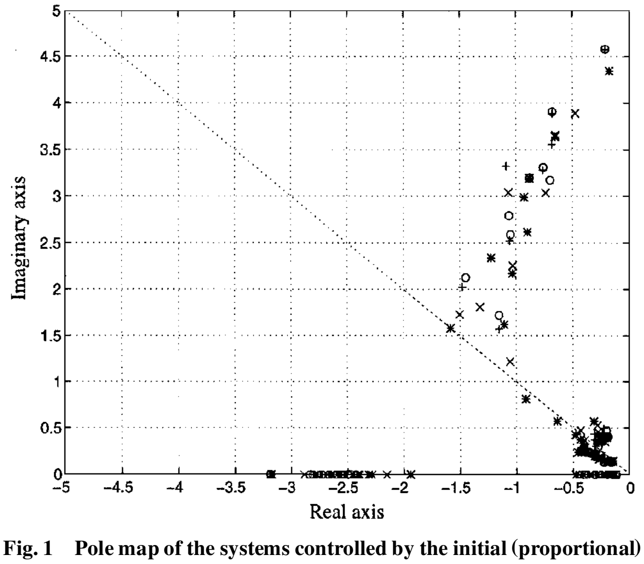

Fig. 1 Pole map of the systems controlled by the initial (proportional) feedback.

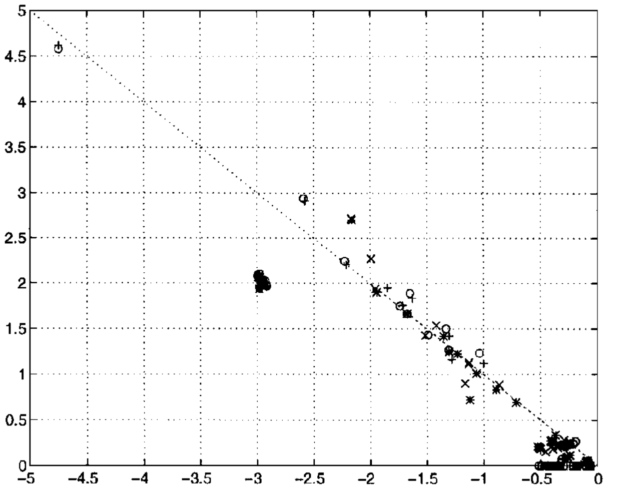

Fig. 2 Pole map of the systems controlled by the final (dynamic) feedback.

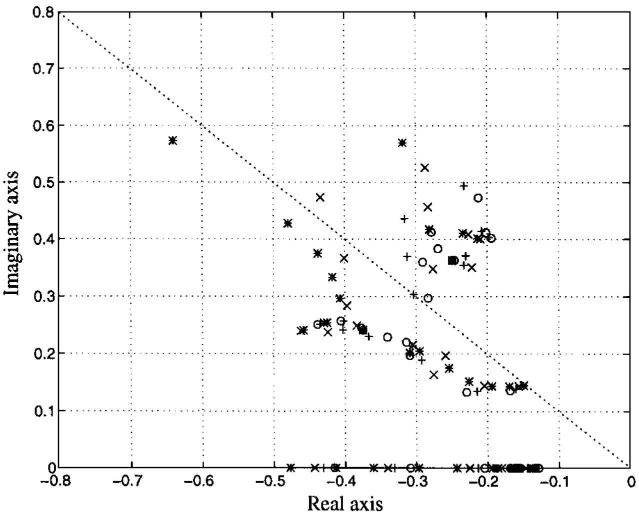

Fig. 3 Zoom of the pole map of the systems controlled by the initial (proportional) feedback.

Therefore, for some column vector $\boldsymbol{\eta}_i \in \mathbb{C}^m$,

$$\boldsymbol{v}_i = V(\lambda_i)\boldsymbol{\eta}_i, \qquad \boldsymbol{w}_i = W(\lambda_i)\boldsymbol{\eta}_i \tag{30}$$

The vector $\boldsymbol{v}_i$ is the orthogonal projection of a given vector $\boldsymbol{v}_{i0}$ if $\boldsymbol{\eta}_i$ is given by

$$\boldsymbol{\eta}_i = \left[V(\lambda_i)^* V(\lambda_i)\right]^{-1} V(\lambda_i)^* \boldsymbol{v}_{i0} \tag{31}$$

where the asterisk stands for both conjugation and transposition.

Now, a multimodel design step is described. The dynamic feedback we look for is as defined in Eq. (20). The choices of the numerator and denominator are discussed separately.

B. Choice of the Denominator Coefficients

The degree of the denominator (q) is related to the number of degrees of freedom; $q + 1$ degrees of freedom appear as coefficients of each numerator. Therefore, the total number of available degrees of freedom is equal to $(q + 1)mp$.

The poles of the denominator must not be too close to the imaginary axis because otherwise they might become more readily unstable in closed loop. In practice, the motion of these poles is a good measure of the realism of the design requirements. Recall that, if no open-loop poles are moved, of course the poles of the denominator are not modified, so that by a continuity argument, if the design requirements are reasonable the stability of these poles is not troublesome. Rolloff properties must be considered for this choice.

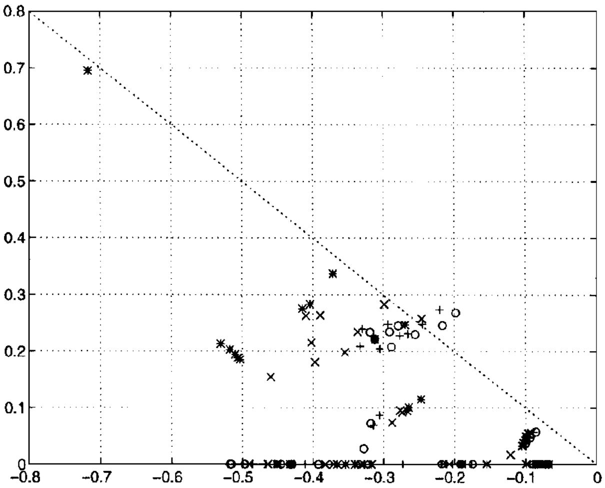

Fig. 4 Zoom of the pole map of the systems controlled by the final (dynamic) feedback.

C. Computation of the Numerator

To illustrate this point, let us consider a simple example; the generalization is straightforward. Assume that the system is modeled by two linear models (possibly controlled by a preliminary feedback) (A_1, B_1, C_1, D_1) and (A_2, B_2, C_2, D_2). For model 1 (respectively, model 2) the open-loop pole $\lambda_{1\text{ol}}$ (respectively, $\lambda_{2\text{ol}}$) must be shifted to $\lambda_{1\text{cl}}$ (respectively, to $\lambda_{2\text{cl}}$). For this assignment we shall proceed as follows.

1) Compute the open-loop eigenvectors $\boldsymbol{v}_{1\text{ol}}$ (relative to A_1) and $\boldsymbol{v}_{2\text{ol}}$ (relative to A_2) corresponding to the open-loop eigenvalues $\lambda_{1\text{ol}}$ and $\lambda_{2\text{ol}}$.

2) Find the vectors $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ corresponding to orthogonal projections, then

$$\boldsymbol{v}_{1\text{cl}} = V_1(\lambda_{1\text{cl}})\boldsymbol{\eta}_1, \qquad \boldsymbol{v}_{2\text{cl}} = V_2(\lambda_{2\text{cl}})\boldsymbol{\eta}_2 \tag{32}$$

$$\boldsymbol{w}_{1\text{cl}} = W_1(\lambda_{1\text{cl}})\boldsymbol{\eta}_1, \qquad \boldsymbol{w}_{2\text{cl}} = W_2(\lambda_{2\text{cl}})\boldsymbol{\eta}_2 \tag{33}$$

where V_1, V_2, W_1, and W_2 are as in Eq. (29) but (A, B) are replaced by (A_1, B_1) or (A_2, B_2) and $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ are obtained in a similar way from Eq. (31).

3) Add some linear constraints relative to the coefficients b_{ijk}, for example, if the gain $K(s)$ must be strictly proper, or if some coefficients must be constant, equal to zero, etc., and define the linear constraints corresponding to eigenstructure assignment:

$$\begin{aligned} K(\lambda_{1\text{cl}})(C_1\boldsymbol{v}_{1\text{cl}} + D_1\boldsymbol{w}_{1\text{cl}}) &= \boldsymbol{w}_{1\text{cl}} \\ K(\lambda_{2\text{cl}})(C_2\boldsymbol{v}_{2\text{cl}} + D_2\boldsymbol{w}_{2\text{cl}}) &= \boldsymbol{w}_{2\text{cl}} \end{aligned} \tag{34}$$

A closed form of these constraints follows easily from Eq. (26).

4) If there are more degrees of freedom than linear equations, consider a criterion quadratic in the coefficients b_{ijk} (λ_i are fixed a priori). Then solve a linear quadratic problem.

5) Find a minimal realization of $K(s)$ if a state-space representation of the feedback is needed.

V. Example: Design of the Longitudinal Autopilot of a Large Transport Aircraft

A. Problem Formulation

The proposed approach is applied to the design of the longitudinal autopilot of the Research Civil Aircraft Model in landing phase. A six-degree-of-freedom nonlinear model is described in detail in Ref. 16. Given here are the results relative to the inner (stabilizing) longitudinal loop. Four standard longitudinal aircraft state variables plus state variables modeling delays, engine, and actuator dynamics have been considered. Two inputs, tailplane angle δq and sum of both engine thrusts δ_{TH}, and four outputs, vertical speed W_V, inertial longitudinal velocity V, pitch rate q, and vertical load factor n_z, have been used. The complete problem including the lateral channel treatment, the outer (tracking) loops, and nonlinear simulations is considered in Ref. 19. We have to fulfill performance criteria.

1) Total velocity V response (settling time) is within 45 s, i.e., time constant smaller than 15 s.

2) Vertical speed W_V response (settling time) is within 20 s, i.e., time constant smaller than 6.67 s.

3) Cross coupling between V and W_V should be as follows. The peak variation of V must be less than 1 m/s for a step demand of W_V: $W_{V,c} = 4.2$ m/s; the peak variation of W_V must be less than 0.7 m/s for a step demand of V: $V_c = 13$ m/s.

4) Overshoots should be less than 5%, i.e., damping ratio lower than 0.7 should be avoided.

In addition there are robustness criteria. The performance requirements must be satisfied within the flight envelope defined as follows.

1) Mass (m) variations are $100{,}000\,\text{kg} \le m \le 150{,}000\,\text{kg}$.

2) Velocity (V) variations are $1.23\,V_{\text{stall}}(m) \le V \le V_{\max}$. For example,

$$V_{\text{stall}}(100{,}000\,\text{kg}) \approx 60\ \text{m/s}$$

$$V_{\text{stall}}(150{,}000\,\text{kg}) \approx 70\ \text{m/s}, \qquad V_{\max} \approx 90\ \text{m/s}$$

3) Center of gravity (CoG_x horizontal, CoG_z vertical) variations are

$$0.15\,\bar{c} \le CoG_x \le 0.31\,\bar{c}, \qquad 0.00\,\bar{c} \le CoG_z \le 0.21\,\bar{c}$$

with $\bar{c}$ the mean aerodynamic chord.

4) The control signal time delay variations are between 50 and 100 ms.

B. Design Procedure

Two models will be considered. We choose the most distant ones to improve the efficiency of the multimodel technique. Note that, if design objectives somewhat different are applied to two models that are too close to each other, the results are usually very bad. To explain this point, assume that, in the limit, the two models become the same one; from the mathematical point of view this means that we obtain a set of noncompatible equations. Moreover, considering models that are too close, it is not easy to define compatible design objectives. This drawback inherent to all multimodel techniques is alleviated for distant models. Considering this remark, we shall take into account

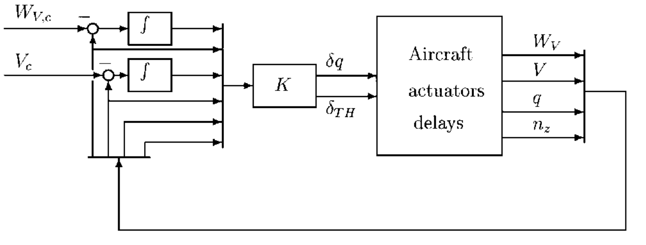

Fig. 5 Controller structure.

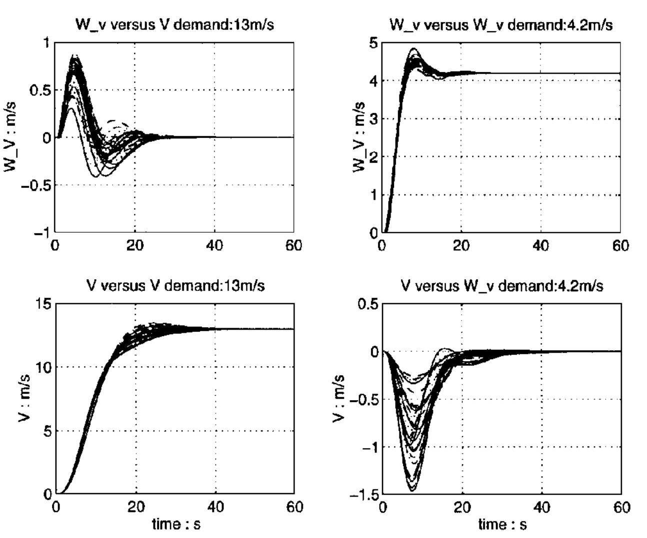

Fig. 6 Step responses of the systems controlled by the initial (proportional) feedback.

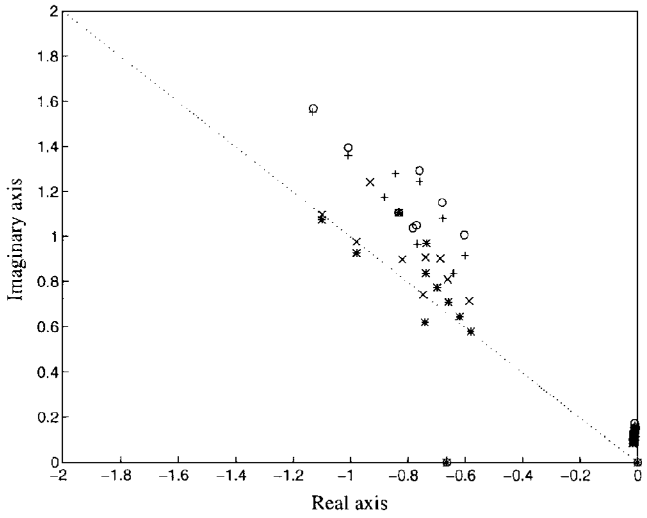

Fig. 7 Open-loop pole map for different configurations.

the two most distant models: model 1 corresponds to high mass and low speed, model 2 corresponds to low mass and high speed; other parameters are less important. More precisely we have for model 1: $m = 150\,t$, $CoG_x = 0.31\,\bar{c}$, $CoG_z = 0.21\,\bar{c}$, $V = 70$ m/s, and time delay = 0.075 s; and for model 2: $m = 100\ t$, $CoG_x = 0.15\ \bar{c}$, $CoG_z = 0.0\ \bar{c}$, $V = 90$ m/s, and time delay = 0.075 s.

1. First Step: Initial Proportional Gain Design

The first step is done by a traditional eigenstructure assignment relative to model 1. From the Signal Flow Diagram,[20] three modes are associated with W_V and one with V. The integrators in Fig. 5 augment the number of poles. Four poles are chosen to be decoupled from V and two decoupled from W_V (this choice was made using the Naslin rule[21] considering the desired settling time given in Sec. V.A). Because the eigenvalues to be assigned are known, it remains to assign eigenvectors and to compute the initial gains (denoted K_0). This step is traditional and detailed in several references.[4,5,7]

2. Multimodel Analysis

To assess robustness and performance, a bank of 27 linearized models covering all of the flight domain is considered. These models

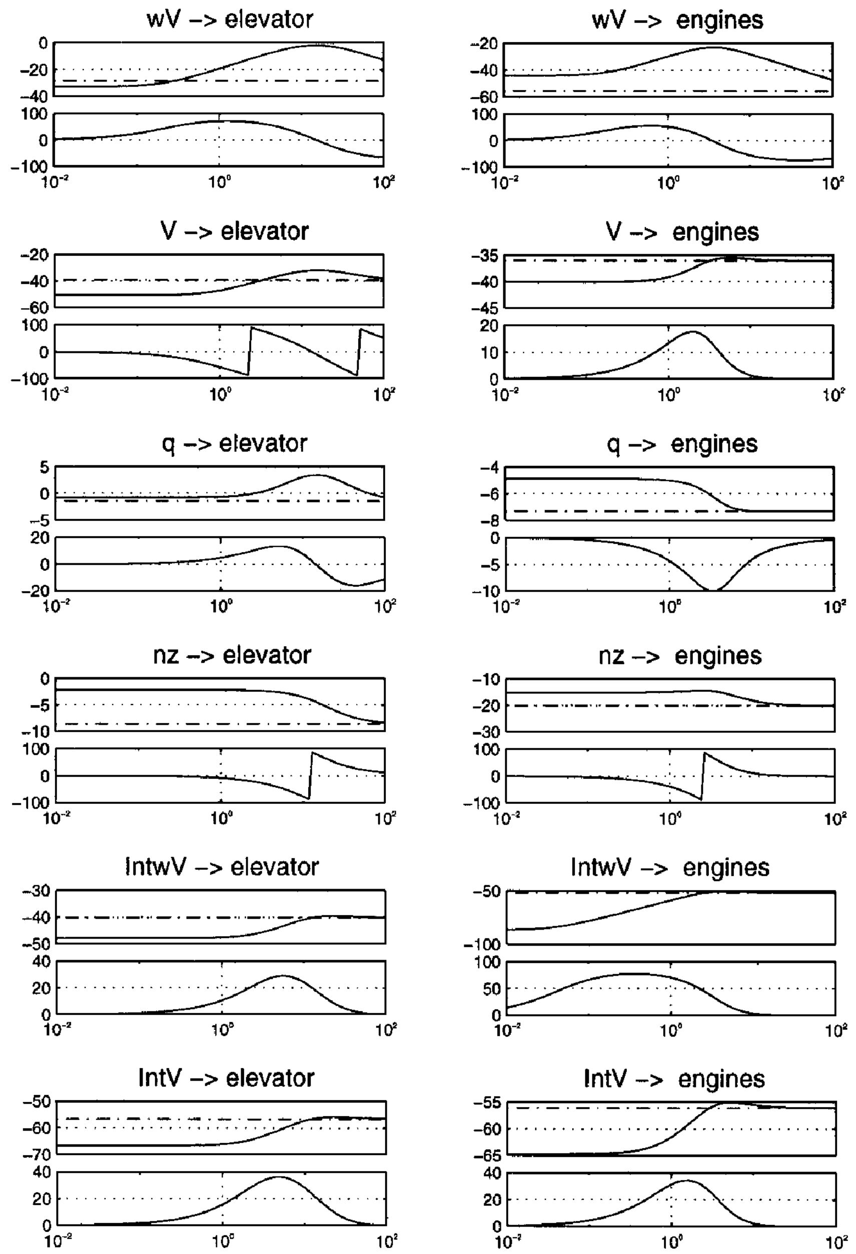

Fig. 8 Bode diagram of the controller.

are controlled by the initial feedback, and the poles are plotted together in Figs. 1 and 3. Step responses to demands of V and W_V showing cross couplings are also given. Figure 6 shows the performance (settling time and decoupling) for the different configurations of the system. The comparison of Fig. 1 with the open-loop pole map in Fig. 7 reveals that, whereas low-frequency eigenvalues are shifted to the left, high-frequency eigenvalues become less damped. Moreover, while the output behavior of the commanded quantities W_V and V are acceptable, decoupling must be improved: ΔW_V exceeds the limits and damping ratio becomes less than 0.5. Clearly, this controller cannot deal with all configurations. Inspection of Figs. 1, 3, and 6 permitted us to check that the worst case was model 2.

3. *Second Step: Multimodel Dynamic Gain Design*

The second step consists of the design of a dynamic controller able to stabilize and robustify the initial gain. The technique of Sec. IV is applied. Figure 1 shows that some poles of Fig. 7 have moved to the right after closing the first loop corresponding to the initial controller. We will attempt to shift back these poles with the objective of keeping the good performance relative to model 1.

a. Choice of the denominator coefficients. Expected rolloff properties are the main constraints for this choice. Concerning the order, an initial value equal to two is chosen (it will turn out to be sufficiently high to have enough degrees of freedom to solve the system of equations involved in the numerator computation).

b. Computation of the numerator. Applying the initial feedback, the eigenvalues of models 1 and 2 are as given in Table 1. Table 1 shows that we have to shift to the left two pairs of eigenvalues for model 2 ($-0.1914 \pm 0.4024i$ and $-0.0002 \pm 4.4192i$). They are assigned to have a damping ratio of 0.7. To avoid undesirable degradation of the initial solution, some first model eigenvalues (the ones assigned by the initial proportional gain) are fixed to their initial positions (good damping ratio and good time response). These assignments lead to the resolution of nine equations similar to those in Eq. (34) (with only six measurements). Furthermore, some constraints on the coefficient b_{ijk} are added to ensure the controller rolloff (see Fig. 8). Unused degrees of freedom are used to minimize the following criterion:

$$J = \sum_i \|K(s_i) - K_0\|_F$$

[$M = -K_0$ and $N = I$ in Eq. (27)], which is quadratic in the coefficients b_{ijk}. The frequencies $s_i = j\omega_i$ are chosen as being different from the assigned eigenvalue frequencies to avoid conflicting objectives.

The minimal realization of the resulting gain $K(s)$ leads to a fourth-order controller.

4. *Multimodel Analysis*

The results are shown in Figs. 2, 4, and 9. We can see that all eigenvalues for all flight configurations are well damped. The time responses show that the pole's nature is satisfied (V slower than W_V). The vertical speed is very satisfactory. The inertial velocity time response is within desired bounds, and the coupling between V and W_V satisfies cross-coupling specifications. It was necessary

Table 1 Eigenvalues of models 1 and 2 controlled by the initial proportional feedback

Model 1 eigenvalues	Model 2 eigenvalues
-2.016	-3.1322
-0.4734	-0.1291
$-0.9972 + 0.9022i$	$-0.0002 + 4.4192i$
$-0.9972 - 0.9022i$	$-0.0002 - 4.4192i$
$-0.6327 + 0.5725i$	$-0.1914 + 0.4024i$
$-0.6327 - 0.5725i$	$-0.1914 - 0.4024i$
$-0.15 + 0.1465i$	$-0.4391 + 0.2525i$
$-0.15 - 0.1465i$	$-0.4391 - 0.2525i$
$-30.73 + 21.41i$	$-31.3 + 26.09i$
$-30.73 - 21.41i$	$-31.3 - 26.09i$
$-30 + 17.32i$	$-30 + 17.33i$
$-30 - 17.32i$	$-30 - 17.33i$

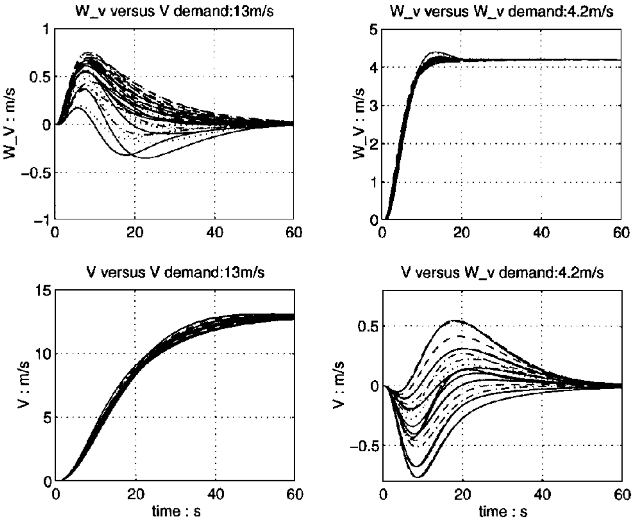

Fig. 9 Step responses of the systems controlled by the final (dynamic) feedback.

to augment the settling time corresponding to the inertial velocity to its maximum allowed limit to decrease engine activity. It is shown in Ref. 19 that, in the presence of atmospheric disturbances, control surface and engine activity is within (additional[16]) specifications. Considering these results, it is not necessary to perform a third step.

VI. Conclusion

We have derived and analyzed the longitudinal control law of a large transport aircraft. The flight domain consists of the glidepath and localizer beam capture and landing phase. In practice, gain scheduling is used for designing such an autopilot. The proposed dynamic feedback has a low order and does not need to be scheduled. The degrees of freedom introduced by the dynamic feedback replace advantageously those introduced by scheduling. In fact, this is not a surprise because the most important parameter for robustness is the mass. It is also known that mass variation induces changes in the dynamics; therefore, the proposed autopilot can be viewed as automatically scheduled with respect to mass variations via frequency variations. The proposed results are better than those obtained by gain scheduling with respect to the speed. In fact, when gain scheduling is used, the scheduled value is the speed. Consequently, each controller has to be robust to the mass variations for each speed. This property is not easily obtained.

Some care must be exercised to avoid conflicting objectives. This problem is inherent to multimodel design techniques. Some guidelines (eigenvector projections) are given to cope with this problem.

The same approach had been considered in a more general setting[19] where outer and inner (including lateral law) loops are designed and the analysis is performed using a nonlinear six-degree-of-freedom simulation tool. Engine failure, change of flight path angle, 90-deg turn, and windshear are considered to assess the quality of the design. It is shown in Ref. 19 that the proposed control law satisfies all design criteria including engine and control surface activity.

The key point of the proposed approach was the transformation of a well-known equation leading to proportional feedback into a similar equation permitting dynamic feedback design. This equation can be solved in various ways. We are investigating improvements to find directly state-space representations. An alternative technique is also being tested considering observer-based dynamic feedback.[17] The advantage of the observer-based gains is that the closed-loop dynamics introduced by the feedback can be fixed a priori.

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